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# Quark Masses in Terms of Gauge Constants and Cabibbo Angle

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Fritzsch type of mass matrices for the $2 \times 2$ case and appropriate Lagrangians enable the choice of Yukawa constants of the Lagrangians in terms of the gauge constants. The mass matrices for the four quarks are shown to be proportional to $V_L$. The Cabibbo angle is computed to be $13°36'$.

**KEY WORDS:** standard model; Yukawa coupling constants; Fritzsch mass matrix; Lagrangian; Cabibbo mixing angle.

## 1. INTRODUCTION

In the standard model, the Yukawa coupling constants of the Lagrangian which generate Fermion masses through Higgs scalars or multiplets have no constraints on them to relate them to the gauge constants. The Yukawa constants are arbitrary. Generally these Yukawa constants are put in by hand in the Lagrangian. When such Lagrangians are used to predict experimental results, one obtains the numerical values of these Yukawa constants.

In this paper we choose a mass matrix which is real and symmetric. This is a $3 \times 3$ Fritzsch (1978, 1979; Li, 1979) mass matrix. We reduce this mass matrix for the case of a four-quark model. A Lagrangian is chosen with CP violation (Mohapatra and Senjanovic, 1981; Raju, 1985, 1986, 1997; Mohapatra, 1972; Mohapatra and Pati, 1975; Pais, 1973; see also, Taylor, 1976). The Yukawa constants of the Lagrangian *s* are so chosen so that the resulting $2 \times 2$ mass matrices are identical to the $2 \times 2$ Fritzsch mass matrices. This choice of the Yukawa constants results in eigen values for $m_u$, $m_c$, $m_d$ and $m_s$ which agree approximately with the known constituent masses of these quarks. Using these masses we also determine the Cabibbo mixing angle which agrees approximately with experiment. This suggests our choice of the Yukawa constants of the Lagrangians is correct.

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These Yukawa constants contain gauge constants in a complicated way but the mass matrix is proportional to the VEV, $V_L$.

The paper is organized in the following way. Section 2 contains Fritzsch mass matrices and the orthogonal matrices that diagonalize these matrices. In Section 3 we reduce these $3 \times 3$ Fritzsch mass matrices to the $2 \times 2$ mass matrices relevant to a four-quark case. Using the well-known definition for mixing, we derive a formula for the Cabibbo mixing angle. Section 4 contains the Lagrangians for the $(u, c)$ case. In Section 5 the Lagrangians for the $(d, s)$ case are presented. Section 6 contains discussion of our results.

## 2. FRTIZSCH TYPE OF MASS MATRICES

The Fritzsch Ansatz (1978, 1979; Li, 1979) for the mass matrix states that only the heaviest generation has a diagonal element and all other lighter masses arise through mixings between neighboring families. We have for $a = u, d$,

$$F^a = \begin{pmatrix} 0 & A^a & 0 \\ A^a & 0 & B^a \\ 0 & B^a & C^a \end{pmatrix} \tag{1}$$

where $F^a$ is known as Fritzsch mass matrix. It is real and symmetric. The nonzero elements $A$, $B$, $C$ of the Fritzsch mass matrix can be expressed in terms of the three eigen values $m_1$, $-m_2$, and $m_3$ by equating the invariants (e.g. the trace and the determinant).

$$A = \sqrt{\frac{m_3 m_2 m_1}{m_3 - m_2 + m_1}}, \tag{2}$$

$$B = \sqrt{\frac{(m_3 - m_2)(m_3 + m_1)(m_2 - m_1)}{m_3 - m_2 + m_1}} \tag{3}$$

and

$$C = (m_3 - m_2 + m_1) \tag{4}$$

For the sake of completeness we reproduce here the elements of the orthogonal matrix that diagonalizes the $3 \times 3$ Fritzsch mass matrix.

$$O_{11} = \sqrt{\frac{m_3 m_2 (m_3 - m_2)}{(m_3 - m_1)(m_2 + m_1)(m_3 - m_2 + m_1)}},$$

$$O_{12} = \sqrt{\frac{m_1 (m_3 - m_2)}{(m_3 - m_1)(m_2 + m_1)}},$$

$$O_{13} = -\sqrt{\frac{m_1(m_2 - m_1)(m_3 + m_1)}{(m_3 - m_1)(m_2 + m_1)(m_3 - m_2 + m_1)}},$$

$$O_{21} = -\sqrt{\frac{m_3 m_1(m_3 + m_1)}{(m_3 + m_2)(m_2 + m_1)(m_3 - m_2 + m_1)}},$$

$$O_{22} = \sqrt{\frac{m_2(m_3 + m_1)}{(m_3 + m_2)(m_2 + m_1)}},$$

$$O_{23} = -\sqrt{\frac{m_2(m_3 - m_2)(m_2 - m_1)}{(m_3 + m_2)(m_2 + m_1)(m_3 - m_2 + m_1)}},$$

$$O_{31} = \sqrt{\frac{m_2 m_1(m_2 - m_1)}{(m_3 + m_2)(m_3 - m_1)(m_3 - m_2 + m_1)}},$$

$$O_{32} = \sqrt{\frac{m_3(m_2 - m_1)}{(m_3 + m_2)(m_3 - m_1)}},$$

and

$$O_{33} = \sqrt{\frac{m_3(m_3 - m_2)(m_3 + m_1)}{(m_3 - m_1)(m_3 + m_2)(m_3 - m_2 + m_1)}}. \tag{5}$$

## 3. THE CABIBBO MIXING MATRIX

When there are only four quarks ($u$, $c$, $d$, $s$) the corresponding Fritzsch type of mass matrices can be obtained from the $3 \times 3$ Fritzsch matrices by first setting $m_1 = 0$, and then taking $m_3 = m_2$ and $m_2 = m_1$ in Equations (2)–(4). Thus for the two generation case the Fritzsch type of real symmetric mass matrix is given by

$$M_c^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{m_2 m_1} \\ 0 & \sqrt{m_2 m_1} & m_2 - m_1 \end{pmatrix}, \tag{6}$$

where $a = u$ or $d$ and the subscript $c$ is for Cabibbo. For example for the ($u$, $c$) case the mass matrix is,

$$M_c^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{m_c m_u} \\ 0 & \sqrt{m_c m_u} & m_c - m_u \end{pmatrix}, \tag{7}$$

From the orthogonal matrix $O$ given by Equation (5) we can also obtain the orthogonal matrix $O_c^a$ that diagonalizes (6) in the following way. First put $m_1 = 0$ in Equation (5) and then take $m_3 \to m_2$ and $m_2 \to m_1$. The orthogonal matrix that diagonalizes Equation (6) is thus given by,

$$O_c^a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\dfrac{m_2}{m_2+m_1}} & -\sqrt{\dfrac{m_1}{m_1+m_2}} \\ 0 & \sqrt{\dfrac{m_1}{m_2+m_1}} & \sqrt{\dfrac{m_2}{m_2+m_1}} \end{pmatrix}, \tag{8}$$

where again $a = u$, or $d$. The Cabibbo mixing matrix is given by,

$$V_c = O_c^d \left(O_c^u\right)^{\text{trans}} \tag{9}$$

In $V_c$ there are no complex phases. The only parameter is commonly taken to be the Cabibbo mixing angle $\theta_c$, and we write,

$$V_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_c & -\sin\theta_c \\ 0 & \sin\theta_c & \cos\theta_c \end{pmatrix}, \tag{10}$$

From Equations (8)–(10) it just follows that,

$$\theta_c = \theta_2 - \theta_1 = \left[\tan^{-1}\sqrt{\frac{m_d}{m_s}} - \tan^{-1}\sqrt{\frac{m_u}{m_c}}\right] \tag{11}$$

## 4. THE (u, c) MASS MATRIX

Let the Higgs sector consist of multiplet (Mohapatra and Senjanovic, 1981) $\phi(\frac{1}{2}, \frac{1^*}{2}, 0)$ and $\tilde{\phi} = \tau_2\phi^*\tau_2, (\frac{1}{2}, \frac{1^*}{2}, 0)$ such that,

$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}, \tag{12}$$

where $\kappa$ and $\kappa'$ are real. In addition to the above multiplet, the Higgs sector has the Higgs scalar $\phi_L$ corresponding to the standard model, and another Higgs scalar $\phi_R$ corresponding (Raju, 1985, 1986) to the left-right model with,

$$\langle\phi_L\rangle = V_L, \tag{13}$$

and

$$\langle\phi_R\rangle = V_R, \tag{14}$$

where $V_L$ and $V_R$ are real. Let the Lagrangian for the $(u, d)$ quarks with $\phi$ be,

$$-L_1 = h_1 \bar{Q}_{1L} \phi Q_{1R} + h_2 \bar{Q}_{1L} \tilde{\phi} Q_{1R} + H.C, \tag{15}$$

where,

$$Q_{1L} = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \text{and} \quad Q_{1R} = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \tag{16}$$

From the above Lagrangian we note that the masses of '$u$' and '$d$' quarks are given by,

$$m_u = m_1 = h_1 \kappa + h_2 \kappa', \tag{17}$$

and

$$m_d = m_3 = h_1 \kappa' + h_2 \kappa, \tag{18}$$

The Yukawa coupling constants $h_1$ and $h_2$ are real. In an exactly similar fashion we assume that $\phi$ is also coupled to '$c$' and '$s$' quarks such that,

$$-L_2 = h_3 \bar{Q}_{2L} \phi \bar{Q}_{2R} + h_4 \bar{Q}_{2L} \tilde{\phi} \bar{Q}_{2R} + H.C. \tag{19}$$

where,

$$Q_{2L} = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \text{and} \quad Q_{2R} = \begin{pmatrix} c_R \\ s_R \end{pmatrix} \tag{20}$$

At this stage, the masses of $c$ and $s$ quarks are respectively,

$$m_2 = h_3 \kappa + h_4 \kappa', \tag{21}$$

and

$$m_4 = h_3 \kappa' + h_4 \kappa. \tag{22}$$

The coupling constants $h_3$ and $h_4$ are real.

In addition to the above Lagrangians the $(u, c)$ quarks are also coupled to $\phi_L$ and $\phi_R$ (Raju, 1997). Before taking up this Lagrangian let us note the following information. Given a Dirac field $\psi$, the Hermitian scalar and pseudo scalar $\bar{\psi}\psi$ and $i\bar{\psi}\gamma_5\psi$ have opposite CP and T transformation properties. (In this respect they are unlike the vector and axial vector $\bar{\psi}\gamma_\lambda\psi$, $\bar{\psi}\gamma_\lambda\gamma_5\psi$.) This is the key to CP violation by a Higgs field. The simplest model used a Higgs field $\phi_L$ and a Lagrangian containing,

$$m\bar{\psi}\psi + ia\bar{\psi}\gamma_5\psi\phi_L, \tag{23}$$

where '$a$' is a real coupling constant. This conserves CP if $\phi_L$ is assigned $CP = -1$. But if spontaneous symmetry breaking gives $\phi_L$ a non-zero VEV, $V_L$ then (23) may be written,

$$\left(m^2 + a^2 V_L^2\right)^{1/2} \bar{\psi}'\psi' + a\bar{\psi}'(\sin\alpha + i\gamma_5 \cos\alpha)\psi'\phi'_L, \tag{24}$$

where,

$$\phi_L = V_L + \phi'_L \quad \text{and} \quad \psi = \exp\left(-\frac{1}{2} i\gamma_5 \alpha\right)\psi', \tag{25}$$

and,

$$\tan\alpha = \frac{aV_L}{m} \tag{26}$$

Vector or axial vector interactions are unaffected by the transformation from $\psi$ to $\psi'$. The CP violation is now caused by the exchange of $\phi'_L$ particles (Mohapatra, 1972; Mohapatra and Pati, 1975; Pais, 1973; Taylor, 1976).

We can implement the above scheme in the case of $(u, c)$ and $(d, s)$ quarks. For $(u, c)$ quarks the Lagrangian is chosen:

$$\begin{aligned} -L_3 = {} & m_1\bar{u}u + m_2\bar{c}c - a_1\bar{u}u\phi_L + a_0\bar{u}c\phi_L + a_0\bar{c}u\phi_L \\ & + ia_L\bar{c}\gamma_5 c\phi_L + ia_R\bar{c}\gamma_5 c\phi_R, \end{aligned} \tag{27}$$

where $a_1$, $a_0$, $a_L$ and $a_R$ are real Yukawa constants. The first two terms are the contributions of Equations (17) and (21). After spontaneous symmetry breaking and due to the following transformations and restrictions,

$$c = \exp\left(-i\gamma_5 \frac{\alpha_1}{2}\right) c', \tag{28}$$

$$u = \exp\left(-i\gamma_5 \frac{\alpha_2}{2}\right) u', \tag{29}$$

and

$$m_1 = a_1 V_L \quad \text{and} \quad \alpha_1 + \alpha_2 = 0, \tag{30}$$

We can write the Lagrangian $L_3$ in the following way:

$$\begin{aligned} -L_3 = {} & o\bar{u}'u' + a_0 V_L \bar{u}'c' + a_0 V_L \bar{c}'u' + \left[m_2^2 + (a_L V_L + a_R V_R)^2\right]^{1/2} \bar{c}'c' \\ & - a_1 \cos\alpha_1 \bar{u}'u'\phi'_L + a_0\bar{u}'c'\phi'_L + a_0\bar{c}'u'\phi'_L - ia_1 \sin\alpha_1 \bar{u}'\gamma_5 u'\phi'_L \\ & + a_L\bar{c}'[i\gamma_5 \cos\alpha_1 + \sin\alpha_1]c'\phi'_L + a_R\bar{c}'[i\gamma_5 \cos\alpha_1 + \sin\alpha_1]c'\phi'_R \end{aligned} \tag{31}$$

To avoid constant terms like $\bar{c}'\gamma_5 c'$ in the above Lagrangian we require that,

$$\tan\alpha_1 = \frac{(a_L V_L + a_R V_R)}{m_2}, \tag{32}$$

In addition $\alpha_1 + \alpha_2 = 0$ ensures the absence of terms like $\bar{u}'\gamma_5 c'\phi'_L$ and their Hermitian conjugates. The choice $m_1 = a_1 V_L$ is required to obtain a $2 \times 2$ Fritzsch type of mass matrix as in Equation (7). Also the Yukawa constant $a_0$ is chosen in the following way so as to yield the off-diagonal element of the mass matrix as in Equation (7).

$$a_0 = \sqrt{a_1 \left[ \frac{g_L}{\sqrt{2}} a_2 \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2} \right]^{1/2}} \tag{33}$$

The Yukawa constant $a_2$ appears in the Lagrangian for $(d, s)$ quarks (to be discussed in the next section). Moreover we will show there $a_2 V_L = m_d = m_3$. Whenever $m_d$ occurs in the $(u, c)$ mass matrix, we found that it should be replaced by $m'_d$ where $m'_d = m_d \frac{(g_v/g_A)^4_{dsb}}{2}$.

With this Ansatz only we obtain expressions for constituent masses of the $(u, c)$ quarks that are in agreement with the known information. For this reason $\frac{(g_v/g_A)^4_{dsb}}{2}$ appears in Equation (33). Of course a similar Ansatz works well for the $(d, s)$ quarks also (see next section). Here,

$$\left(\frac{g_v}{g_A}\right)^4_{dsb} = \left(-1 + \frac{4}{3}\sin^2\theta_\omega\right)^2 \tag{34}$$

For the $(u, c)$ quarks, the mass matrix is now given by Equation (31),

$$M^u_c = \begin{pmatrix} 0 & a_0 V_L \\ a_0 V_L & \sqrt{m_2^2 + (a_L V_L + a_R V_R)^2} \end{pmatrix} \tag{35}$$

The off-diagonal element $a_0 V_L$ is given by,

$$a_0 V_L = \sqrt{a_1 V_L \left[ \frac{g_L}{\sqrt{2}} V_L a_2 V_L \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2} \right]^{1/2}},$$

$$= \sqrt{m_u [M_{\omega L} m'_d]^{1/2}} \tag{36}$$

To bring in $M_{\omega L}$, the gauge constant of $SU(2)_L$, $\frac{g_L}{\sqrt{2}}$ is chosen in $a_0$. Similarly $a_1$ and $a_2$ appear in $a_0$ to bring in $m_u$ and $m_d$. If we ignore the trivial elements of the $3 \times 3$ matrix of Equation (7), $M^u_c$ has the desired form now in (35).

The diagonal term in (35) contains three unknowns, $a_L$, $a_R$ and $V_R$. There is no loss of generality if we re-express the three unknowns in terms of $m'_d$, $m_u$ and $m_2$, which are also as of now, are unknown. To this end we take,

$$a_L^2 V_L^2 \left(1 + \frac{a_R V_R}{a_L V_L}\right)^2 = [(m'_d M_{\omega L})^{1/2} - m_u]^2 - m_2^2 \tag{37}$$

Collecting all the terms, the 2 × 2 mass matrix $M_c^u$ now reads,

$$M_c^u = V_L \begin{pmatrix} 0 & \sqrt{a_1 \left[ \frac{g_L}{\sqrt{2}} a_2 \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2} \right]^{1/2}} \\ \sqrt{a_1 \left[ \frac{g_L}{\sqrt{2}} a_2 \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2} \right]^{1/2}} & \left[ a_2 \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2} \frac{g_L}{\sqrt{2}} \right]^{1/2} - a_1 \end{pmatrix} \tag{38}$$

Except $a_1$ and $a_2$, all the Yukawa constants in this mass matrix are all gauge constants of the standard model. These gauge constants appear in a complicated form. The mass matrix $(M_c^u)(M_c^u)^+$ is diagonalized by Equation (8). This orthogonal matrix is given by,

$$O_c^u = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix}, \tag{39}$$

where

$$\tan\theta_1 = \sqrt{\frac{m_u}{m_c}} = \sqrt{A_1} \tag{40}$$

Here,

$$A_1 = \frac{a_1}{\left[ a_2 \frac{g_L}{\sqrt{2}} \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2} \right]^{1/2}} \tag{41}$$

In Equations (40) or (41) if we set $a_1 = a_2$ (which is equivalent to $m_u = m_d$), then,

$$\tan\theta_1 = \sqrt{\left( \frac{a_1}{\frac{g_L}{\sqrt{2}} \frac{\left(\frac{g_v}{g_A}\right)^4_{dsb}}{2}} \right)^{1/2}} \tag{42}$$

The eigen values of $(M_c^u)(M_c^u)^+$ are,

$$m_u^2 = (a_1 V_L)^2, \tag{43}$$

and

$$m_c^2 = m_d'/M_{WL} \tag{44}$$

If $m_u = m_d = 0.3\,\text{GeV}$, and $M_{WL} = 80\,\text{GeV}$, then $m_c = 1.69\,\text{GeV}$. This agrees very approximately with the known constituent mass of the C-quark. Here we took $\sin^2\theta_\omega = 0.2254$ (Raju, 1997).

For $m_u = 0.3\,\text{GeV}$, the Yukawa Constant $a_1$ is given by,

$$0.3\,\text{GeV} = (a_1 V_L) = \frac{a_1 M_{WL}}{(g_L/\sqrt{2})},$$

and hence,

$$a_1 = 0.00265 g_L \tag{45}$$

$$\theta_1 = \tan^{-1}\left(\sqrt{A_1}\right) = 22^\circ 49', \tag{46}$$

where Equation (42) is used for obtaining the angle $\theta_1$.

## 5. THE (*d*, *s*) MASS MATRIX

In the case of (*d*, *s*) quarks the following Lagrangian which is similar in all respects to Equation (27) is chosen.

$$\begin{aligned} -L_4 = {} & m_3\bar{d}d + m_4\bar{s}s - a_2\bar{d}d\phi_L + b_0\bar{d}s\phi_L + b_0\bar{s}d\phi_L \\ & + ib_L\bar{s}\gamma_5 s\phi_L + ib_R\bar{s}\gamma_5 s\phi_R, \end{aligned} \tag{47}$$

where the very first two terms are the contributions of Equations (18) and (22). Again all the Yukawa constants $a_2, b_0, b_L$ are real. In addition,

$$d = \exp\left(-i\gamma_5\frac{\alpha_3}{2}\right)d', \tag{48}$$

$$s = \exp\left(-i\gamma_5\frac{\alpha_4}{2}\right)s'. \tag{49}$$

We also require that,

$$\alpha_3 + \alpha_4 = 0, \tag{50}$$

and

$$m_3 = m_d = a_2 V_L, \tag{51}$$

with

$$\tan\alpha_4 = \left(\frac{b_L V_L + b_R V_R}{m_4}\right). \tag{52}$$

The above requirements ensure the absence of terms like $\bar{d}\gamma_5 s' V_L$ and $\bar{d}'\gamma_5 s'\phi'_L$ and their Hermitian conjugates. The choice $m_3 = a_2 V_L$ is required to obtain a mass matrix that has the desired form as in Equation (6).

With these conditions after spontaneous symmetry breaking the Lagrangian (47) may be written as,

$$\begin{aligned} -L_4 = {} & o\bar{d}'d' + b_0 V_L \bar{d}'s' + b_0 V_L \bar{s}'d' + \left[m_4^2 + (b_L V_L + b_R V_R)^2\right]^{1/2} \bar{s}'s' \\ & - a_2 \cos\alpha_4 \bar{d}'d'\phi_L' + b_0 \bar{d}'s'\phi_L' + b_0 \bar{s}'d'\phi_L' - i a_2 \sin\alpha_4 \bar{d}'\gamma_5 d'\phi_L' \\ & + b_L \bar{s}'[i\gamma_5 \cos\alpha_4 + \sin\alpha_4] s'\phi_L' + b_R \bar{s}'[i\gamma_5 \cos\alpha_4 + \sin\alpha_4] s'\phi_R' \end{aligned} \tag{53}$$

As in the case of $(u, c)$ quarks, here also we require that,

$$b_0 V_L = \sqrt{a_2 V_L \left[\frac{g_L}{\sqrt{2}} V_L a_1 V_L \frac{(g_V/g_A)^4_{uct}}{2}\right]^{1/2}}, \tag{54}$$

and

$$b_L^2 V_L^2 \left(1 + \frac{b_R V_R}{b_L V_L}\right)^2 = (m_u' M_{WL} - m_d)^2 - m_4^2 \tag{55}$$

In Equation (54) $m_u = a_1 V_L$ appears. We replaced $m_u$ by $m_u' = m_u \frac{(g_V/g_A)^4_{uct}}{2}$. This is exactly identical to the Ansatz we followed in the case of the $(u, c)$ mass matrix and the gauge constants there. Only this choice of the Yukawa constants leads to the constituent masses of the $(d, s)$ quarks that agree quite well with experiment.

Collecting all the terms, the mass matrix for the $(d, s)$ case is now given by,

$$M_c^d = V_L \begin{pmatrix} 0 & \sqrt{a_2 \left[\frac{g_L}{\sqrt{2}} a_1 \frac{\left(\frac{g_v}{g_A}\right)^4_{uct}}{2}\right]^{1/2}} \\ \sqrt{a_2 \left[\frac{g_L}{\sqrt{2}} a_1 \frac{\left(\frac{g_v}{g_A}\right)^4_{uct}}{2}\right]^{1/2}} & \left[a_1 \frac{\left(\frac{g_v}{g_A}\right)^4_{uct}}{2} \frac{g_L}{\sqrt{2}}\right]^{1/2} - a_2 \end{pmatrix}. \tag{56}$$

The matrix $(M_c^d)(M_c^d)^+$ is diagonalized by the following orthogonal matrix,

$$O_c^d = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix}, \tag{57}$$

where,

$$\tan\theta_2 = \sqrt{\frac{m_d}{m_s}} = \sqrt{B_2}. \tag{58}$$

Here,

$$B_2 = \frac{a_2}{\left[a_1 \frac{g_L}{\sqrt{2}} \frac{\left(\frac{g_v}{g_A}\right)^4_{uct}}{2}\right]^{1/2}} \tag{59}$$

Moreover,

$$\left(\frac{g_v}{g_A}\right)^4_{uct} = \left(-1 + \frac{8}{3}\sin^2\theta_\omega\right)^2 \tag{60}$$

The eigen values of $(M_c^d)(M_c^d)^+$ matrix are,

$$m_d^2 = (a_2 V_L)^2, \tag{61}$$

and

$$m_s^2 = m_u' M_{WL}, \tag{62}$$

where $m_u' = m_u \frac{(gV/gA)^4_{uct}}{2}$. For $m_u = m_d = 0.3\,\text{GeV}, m_s = 0.55\,\text{GeV}$ which agrees approximately. The exact expressions for $m_c^2$ and $m_s^2$ are given in (Raju, 1997). If $m_u = m_d$, it is equivalent to $a_1 = a_2$ and hence from (58) and (59) we readily note that,

$$\theta_2 = \tan^{-1}\sqrt{\left(\frac{a_1}{\left(g_L/\sqrt{2}\right)\frac{(g_V/g_A)^4_{uct}}{2}}\right)^{1/2}} = 36°25' \tag{63}$$

The above result is obtained by using $a_1 V_L = 0.3\,\text{GeV}$ and $\frac{g_L}{\sqrt{2}} V_L = M_{WL} = 80\,\text{GeV}$ in Equation (63). The Cabibbo angle is given by,

$$\theta_c = \theta_2 - \theta_1 = 13°36'. \tag{64}$$

This is an approximate result. An exact result is given in (Raju, 2000).

## 6. DISCUSSION

In this paper the choice of the Lagrangians $L_3$ and $L_4$ are based on the presumption that there is a CP violation because of the Higgs field whose VEV is not zero. In addition to this requirement, we required that the mass matrix be identical to a Fritzsch type of mass matrix which is real and symmetric. This enabled us to choose the Yukawa constants $a_0$ and $b_0$ in Equations (33) and (54). The masses of the quarks are found here in terms of the gauge constants and VEV $V_L$. The appearance of the ratio $(g_V/g_A)^4$ along with $m_u$ and $m_d$ strengthens our belief that the Higgs scalars or multiplets are bound states of $Z, Z^*, D, D^*$, etc. as explained in (Raju, 1986).

If there is no CP violation, in place of $L_3$ of Equation (28) we can choose the following simple Lagrangian,

$$-L_3 = m_1\bar{u}u + m_2\bar{c}c - a_1\bar{u}u\phi_L + a_0\bar{u}c\phi_L + a_0\bar{c}u\phi_L + a_L\bar{c}c\phi_L + a_R\bar{c}c\phi_R, \quad (65)$$

After spontaneous symmetry breaking and with the following choices,

$$m_1 = a_1 V_L, \quad (66)$$

and,

$$a_L V_L + a_R V_R = \left[(m'_d M_{WL})^{1/2} - m_1\right] - m_2, \quad (67)$$

we obtain a mass matrix for $(u, c)$ quarks given by,

$$M_c^u = \begin{pmatrix} 0 & a_0 V_L \\ a_0 V_L & [(m'_d M_{WL})^{1/2} - m_1] \end{pmatrix}. \quad (68)$$

The Yukawa constant $a_0$ is still given by Equation (33). The eigen values for $m_u^2$ and $m_c^2$ are also given by Equations (43) and (44).

The Lagrangians for $(d, s)$ quarks can also be written down in a similar way. The basic thing is that the elements of the mass matrix are again proportional to the VEV, $V_L$ and the Yukawa constants do contain the gauge constants like $g_L$ and $(g_V/g_A)^4$. In addition the eigen values for the masses do yield the right values for the Cabibbo angle.

Another basic question is why there is Cabibbo mixing at all. The Cabibbo angle will be zero if $A_1$ and $B_2$ of Equations (41) and (59) are equal. This is equivalent to the following relation,

$$\left(\frac{m_d}{m_u}\right)^{3/2} = \frac{(g_V/g_A)^2_{uct}}{(g_V/g_A)^2_{dsb}} = 0.325, \quad (69)$$

where we used $\sin^2\theta_\omega = 0.2254$. The above relation is equivalent to the relation $m_d = 0.47\ m_u$. If the $(u, d)$ quarks are chargeless then Equation (69) implies that they will have the same mass and there will be no Cabibbo mixing. In the case of Leptons, the electron and muon neutrinos have equal mass (Raju, 1986) and there is no Cabibbo like mixing there. Does this suggest any unity in diversity? If the mass $m_d$ is about half that of $m_u$, there would have been no Cabibbo mixing.

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